Stability and criticality in Ornstein-Uhlenbeck processes with heterogeneous temperatures via correlations: a random matrix approach

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The application of random matrix theory (RMT) to the investigation of complex systems has shown how the spectral properties of eigenvalues are efficient probes for stability and universality. The central idea of the RMT approach is to abandon the search for the exact form of the microscopic interaction matrix, introducing instead probability distributions for its elements.

In the context of ecology, Robert May's 1972 paper, named "Will a large complex system be stable?", pioneered the use of spectral analysis to investigate the stability of the fixed points of deterministic linear dynamical systems, highlighting the importance of the empirical spectral density (ESD) of the eigenvalues of the connectivity matrix in the instability transition. In the stochastic counterpart, where the linear system fluctuates around the fixed point, non-trivial correlations between dynamical variables emerge, contrasting with the previous deterministic scenario. In such cases, being a more experimentally accessible quantity, the covariance matrix of the dynamical variables is a good alternative for the connectivity matrix, which is often hard to measure. However, the behavior of its spectral properties across the stability transition remains elusive.

Building on previous studies [1], in this work [2] we propose a RMT ensemble for the connectivity matrix of multivariate Ornstein-Uhlenbeck processes, the simplest class of linear stochastic models. Imposing a reversibility condition, we obtain the explicit dependence of the correlation matrix on the connectivity and diffusion matrices, that respectively control microscopic interactions and thermal fluctuations. This explicit relation stands in contrast to the majority of the previous works that rely on ad hoc models for the covariance matrix (e.g. [3]). We analitically obtain the ESD of the correlation matrix in terms of arbitrary temperature distributions for the microscopic variables, and use it to determine the stability phase diagram. At the critical line, the ESD presents a power law with a temperature independent exponent, suggesting universal critical behavior.

References

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