Power Law Behavior Around Bifurcation Points of 1-D Maps: A Supertracks Approach

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The convergence towards asymptotic states at bifurcation points BPs $r = r_b$ of 1-D mappings (of a free parameter r) presents scaling laws whose characteristic exponents in principle depend on the maps specific non-linearities [1]. Aiming to better understand such comportment, we investigated the logistic-like and sine-like family of maps by studying transcritical, pitchfork, period-doubling and tangent BPs. For so, we employed the supertrack framework, where continuous functions of r are generated, having as the initial condition the 1-D map critical point [2]. From such approach we obtained, by numerical and analytical procedures, four exponents to describe the asymptotic behavior when $r = r_b$ as well as another exponent typifying the case of $r > r_b$. Moreover, we confirmed the universality classes of transcritical and pitchfork BPs proposed in the literature and unveiled novel universality results for period-doubling and tangent BPs. Our findings highlighted the usefulness of the supertracks method, for instance, allowing to uncover universality in dynamical systems and to draw a parallels with critical phenomena [3].

References

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