Nonequilibrium critical exponents of highly connected heterogeneous Ising models with ferromagnetic interactions

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The Ising model plays a fundamental role in the investigation of collective phenomena in complex systems and, due to its simplicity, it stands out as a testing ground for the study of critical behavior. In this work [1], we analytically solve the synchronous dynamics of the ferromagnetic Ising model on an ensemble of networks with arbitrary degree distributions. We also consider arbitrary thermal noise distributions, allowing for non-equilibrium stationary states. We characterize micro and macroscopic dynamics by obtaining equations that describe the dynamics of the global magnetization and of the distribution of local magnetizations. In the high connectivity limit, where the mean degree is very large, our theory retains the effect of topological fluctuations, contrasting with usual mean field theories.

We focus on the characterization of the critical behavior of the ferromagnetic Ising model in the case where the degrees follow a negative binomial distribution. This choice allows for the parametrization of the heterogeneity of the network through a single quantity. Starting by the critical temperature at which the system undergoes a continuous transition from a ferro to a paramagnetic phase, we show that its value is determined by the variance of the degree distribution and by the behavior of the threshold noise distribution near zero. In the homogeneous limit, fluctuations are weak and we recover the phase diagram of the fully connected ferromagnetic Ising model. This equivalence between the homogeneous limit of our model and the fully connected case was later formally proven [2]. Regarding the critical exponents, we show that the degree fluctuations present in heterogeneous networks do not alter the well known values of the fully connected universality class [5]. However, we show that the particular choice for the threshold noise distribution affects the value of the critical exponents, breaking such universality class if the stationary microstate distribution is not a Boltzmann form.

Overall, our work introduces a family of Ising models on random graphs that retain the effect of both topological structure and threshold noise distribution, whose non-equilibrium dynamics can be solved exactly. Our results provide insights on the effects of network structure and stochastic fluctuations on critical phenomena, highlighting their importance on the critical behavior of the ferromagnetic Ising model.

References

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