Nelson's Stochastic Quantization: Investigation of Tunneling Processes

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In 1966, Edward Nelson developed a new formalism based on statistical mechanics to explain quantum dynamics based on the fact that the interaction of the particle with the environment is not negligible [1]. This formalism, known as Nelson's stochastic quantization, postulates that the quantum particle describes a sort of random walk in the form of a conservative diffusion process, with the fluctuation term proportional to Planck's constant. In order to investigate the advantage of the Nelson's formalism in comparison with the standard formulation of quantum mechanics, we analyze the problem of the first passage time of a particle tunneling through a potential barrier. In traditional quantum mechanics, this problem is difficult to address because of the nonexistence of a time operator. In contradistinction, Nelson's stochastic quantization allows us to solve this problem directly, once we can write down the Fokker-Planck equations associated with the quantum brownian particle [2]. In our study, we considered the tunneling of a particle in a double squared well (a squared potential barrier inside a infinity squared well). First, using Nelson's stochastic quantization, the problem was solved analytically in order to found the tunneling time of the particle. Second, the average tunneling time was computed from numerical simulations of several trajectories of a brownian particle subjected to the conditions corresponding to the associated quantum problem. The numerical results were found to be in good agreement with the theoretical prediction. Comparison is also made with results found in the literature for the analogous problem of tunneling in a double-well (quartic) potential [3]. We observed an exponential decay in the tunneling time distribution that appears to be a universal behavior. The problem of tunneling time in ammonia will be also analyzed and discussed.

References

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