Mathematical Models to Explain the Origin of Urban Scaling Laws

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The quest for a theory of cities that could offer a quantitative and systematic approach to managing cities represents a top priority. If such a theory is feasible, then its formulation must be in a mathematical way. As a contribution to organizing the mathematical ideas that deal with such a systematic way of understanding urban phenomena, we review the main theoretical models present in the literature that aim at explaining the origin and emergence of urban scaling. Empirical evidence has shown that some urban variables scale non-linearly with the city population size. More specifically, some socio-economic variables, such as the number of patents, wages and GDP, show a super-linear behaviour with the city's population. On the other hand, infrastructure variables, such as the number of gas stations and length of streets, scale sub-linearly with the city population, generating a scale economy. We intend to identify similarities and connections between different mathematical models used to explain urban scaling and find situations in which different models lead to the same output. Details about these models can be found in [1]. We will present situations where some ideas initially introduced in a particular model can also be introduced in another one, generating more diversification and increasing the scope of the original works. The models treated explain urban scaling from different premises, i.e. from gravity ideas, densification and cites' geometry to a hierarchical organization and social network properties. For instance, the facilitation of mobility and access is identified as an essential ingredient to promoting a scaling economy. We also investigate scenarios in which these different fundamental ideas could be interpreted as similar – where the similarity is likely but not obvious. Furthermore, concerning the gravity model, we propose a general framework that includes all analyzed models as particular cases. We conclude by discussing perspectives of this field and how future research designs and schools of thought can build a more general theory of cities. [1, 2, 3]

References

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