Involution Symmetry Quantification Using Recurrences[1]

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Symmetries are ubiquitous in science, aiding theoretical comprehension by discerning patterns in mathematical models and natural phenomena. This work introduces a method for assessing the extent of symmetry within a time series. We explore both microscopic and macroscopic features extracted from a recurrence plot[2]. By analyzing the statistics of small recurrence matrices[3], our approach delves into micro-scale dynamics, facilitating the identification of symmetric time series segments through diagonal macro-scale structures on a recurrence plot. We validate our approach by successfully quantifying involution symmetries for three-dimensional dynamical models, specifically, order-2 rotational symmetry in the Lorenz'63 model, and inversion symmetry in the Chua circuit. Our quantifier also detects symmetry breaking in the modified Lorenz model for *El Niño* phenomenon. The method can be applied in a versatile manner, not only to three-dimensional trajectories but also to uni-variate time series. Symmetry quantification in time series is promising for enhancing dynamical system modeling and profiling.

References

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